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Finding Low-Order Aerodynamic Models from Experimental Data

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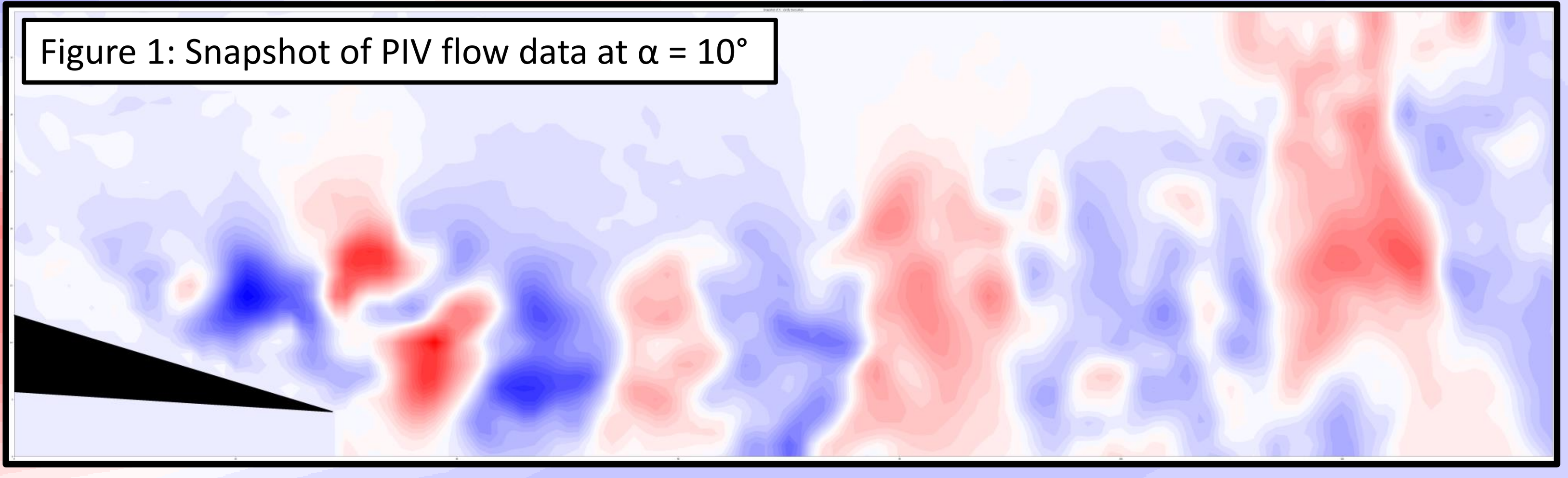
Background

- The flow around wings is incredibly complex due to the presence of large organised structures (von Kármán shedding, Kelvin-Helmholtz instability) and small chaotic motions.
- Reynolds number, angle of attack and free-stream turbulence all impact the vast number of particles in the flow.
- Modal decomposition techniques can extract these structures by energy and frequency for reduced-order modelling. This report compared POD, DMD and BMD decompositions .

Objectives

- Understand POD representation of airfoil flow.
- Identify what structures POD and DMD extract from PIV fluid flow data around a NACA 0018 airfoil at Re = 10250 and α = 0°, α = 10°.
- Compare the structures extracted from the flow using POD to those extracted by DMD.
- Utilise BMD to reveal how structures identified by POD and DMD interact with each other.

Figure 1: Snapshot of PIV flow data at α = 10°



Y	=	Σ	U	V*
Spatial and Temporal PIV Data	=	Singular Values	Left Singular Vector (Spatial Coefficients)	Right Singular Vector (Temporal Coefficients)

Proper Orthogonal Decomposition

- Singular Value Decomposition technique for POD assumes the vectors in the input data matrix are a linear function of space and time coefficients.
- Captures kinetic energy patterns by optimizing the mean square of the field variable.
- Temporal coefficients and energy organised in order of relative energy.
- Symmetric waves initiating at the trailing edge visible.
- Velocity patterns originating above the airfoil show the presence of a shear layer and Kelvin-Helmholtz instability due to boundary layer separation.
- Higher order structures appear as hybrids of multiple flow features summed by POD.
- Many constituent frequencies due to multiple flow structures and complexity of flow interactions.

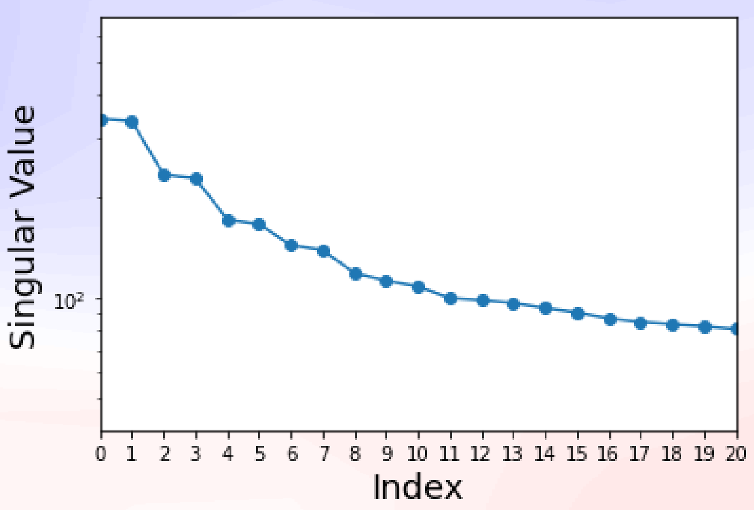


Figure 2: POD singular values for flow around airfoil, α = 10°

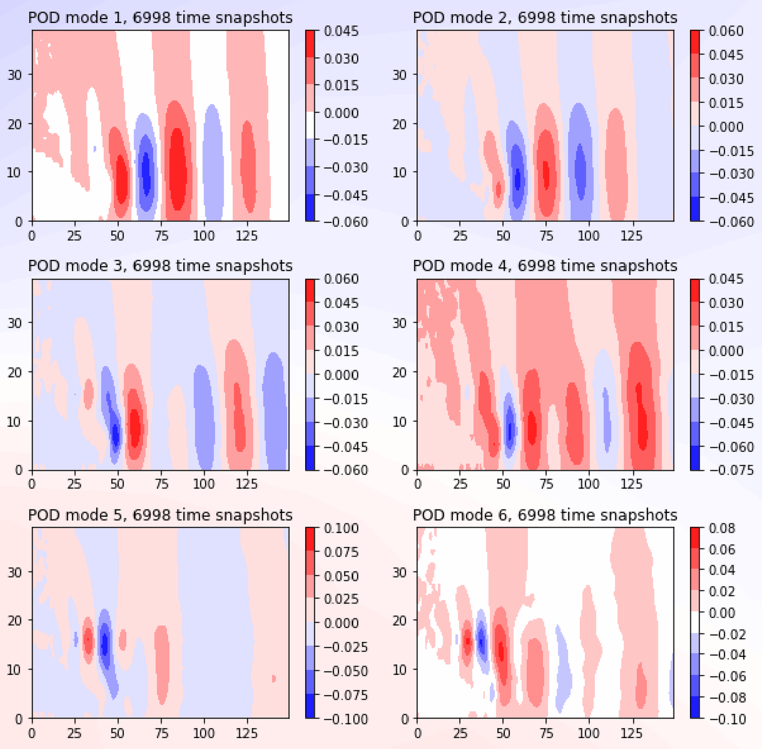


Figure 3: POD modes at α = 10°

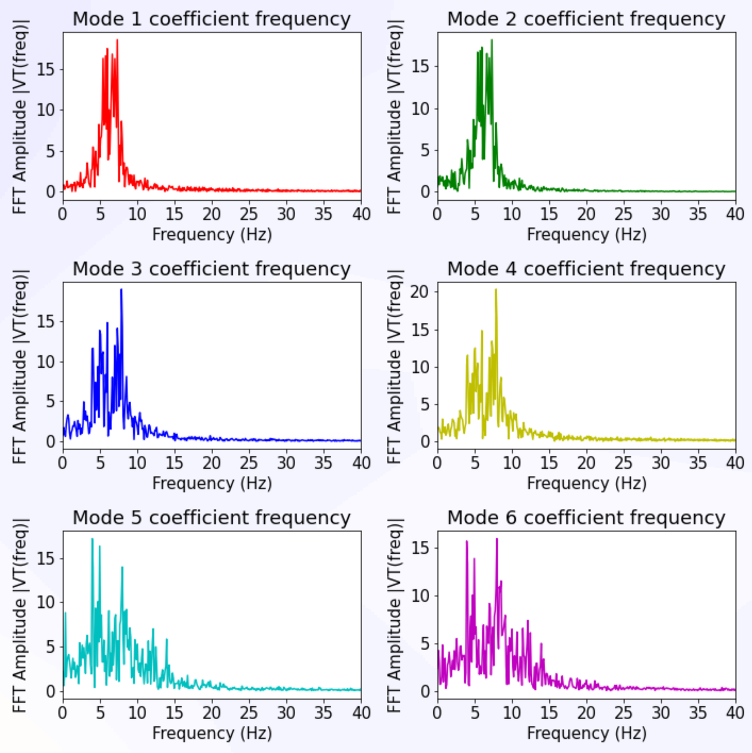


Figure 4: Spectral analysis of temporal coefficients at α = 10°

Dynamic Mode Decomposition

- The eigendecomposition of a best-fit linear operator approximates the dynamics present in the data.
- DMD returns non-orthogonal spatial modes with their corresponding eigenvalues from input data, denoting modal oscillations and growth/decay rates.
- Eigenvalues always appear in complex conjugate pairs of oscillatory modes slightly within the unit circle.
- Structures are dynamically stable, will decay slowly.
- Shear layer with Kelvin-Helmholtz instability and von Kármán vortex street clearly separated.
- Longer wavelengths correspond to lower frequencies, all modes travel with similar convection.
- Evidence of vortex merging visible.
- Clear frequency peaks, well separated flow features.

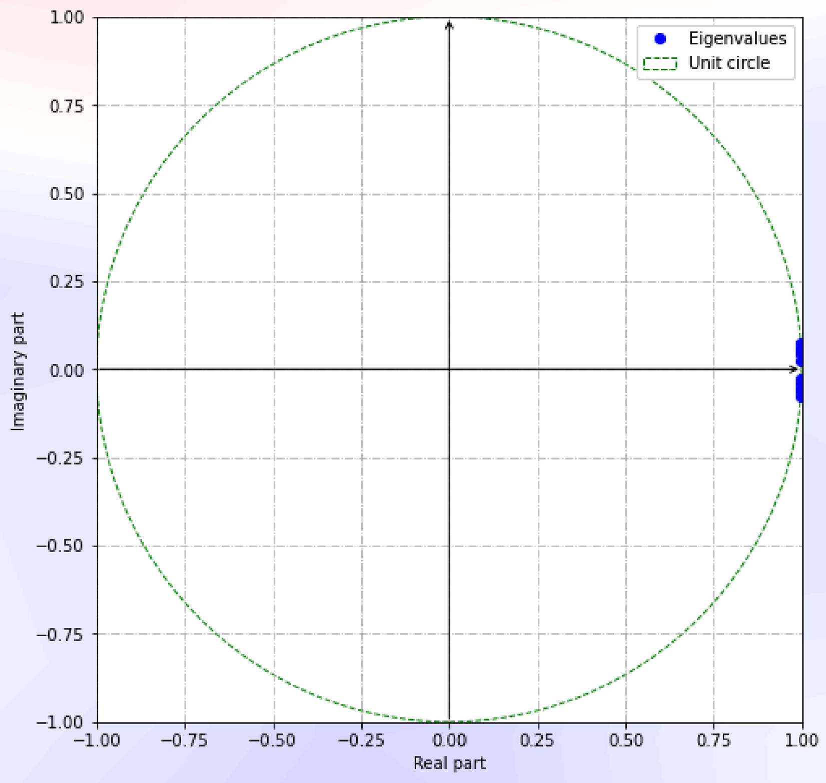


Figure 5: Eigenvalues from DMD at α = 10°

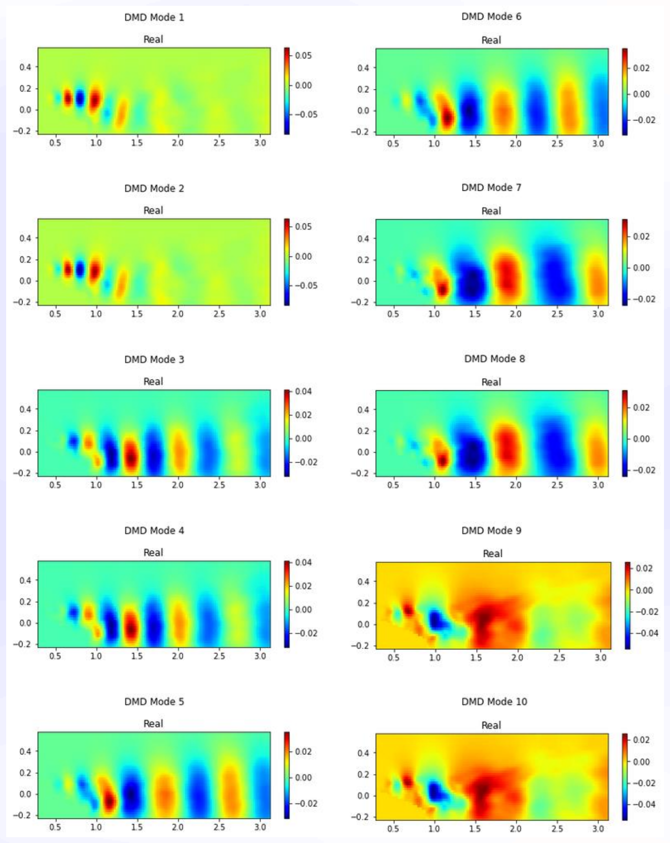


Figure 6: DMD modes at α = 10°

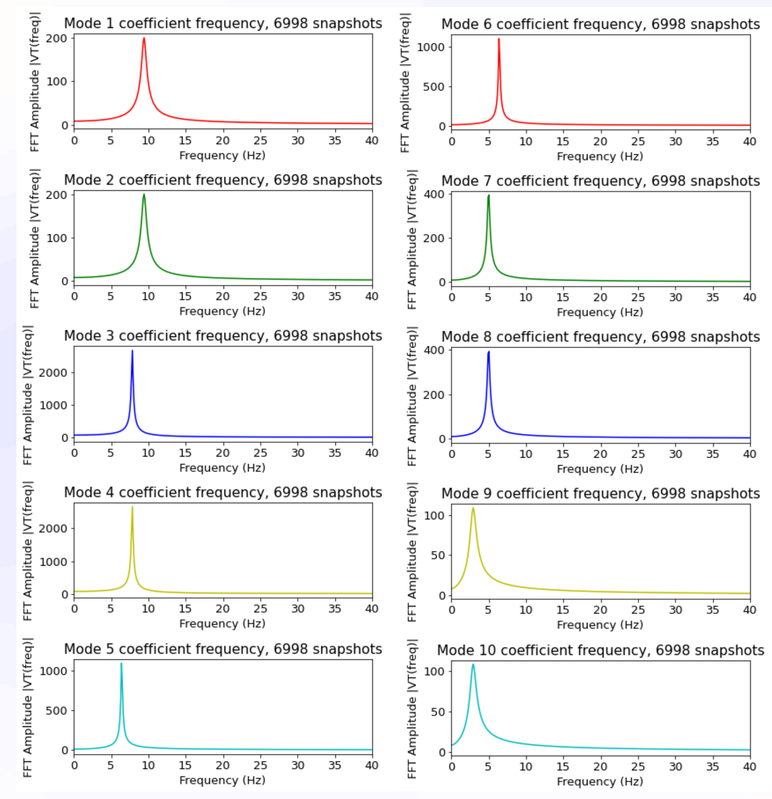


Figure 7: Spectral analysis of DMD modes at α = 10°

$$\left|\frac{\psi_p^{\mathsf{T}}\cdot\psi_d}{\sqrt{\psi_p^{\mathsf{T}}\cdot\psi_p}\cdot\sqrt{\psi_d^{\mathsf{T}}\cdot\psi_d}}\right|$$

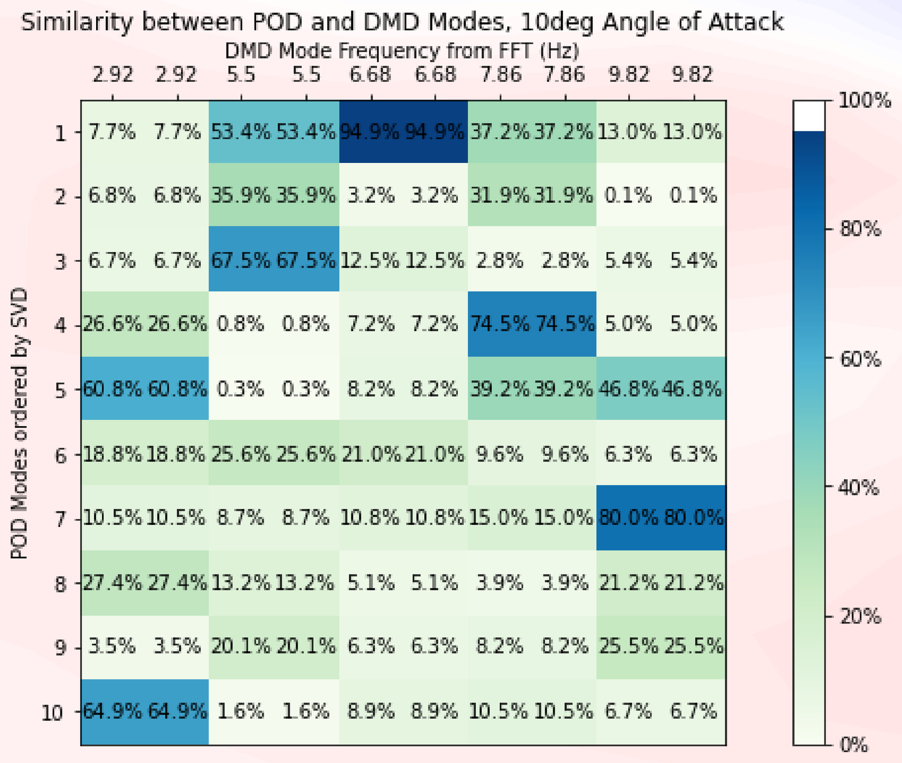


Figure 8: Projections of DMD modes onto POD modes at α = 10°

Projections

- POD modes represented almost entirely by one conjugate DMD mode pair can be attributed to a single flow structure due to their orthogonality.
- Due to increased nonlinear interactions in high-α flow, POD modes at α = 10° are less successful than at α = 0° for discerning single frequency orthogonal structures.
- The fundamental frequencies of the flow change with angle of attack of the airfoil, leading to differences in shedding pattern.

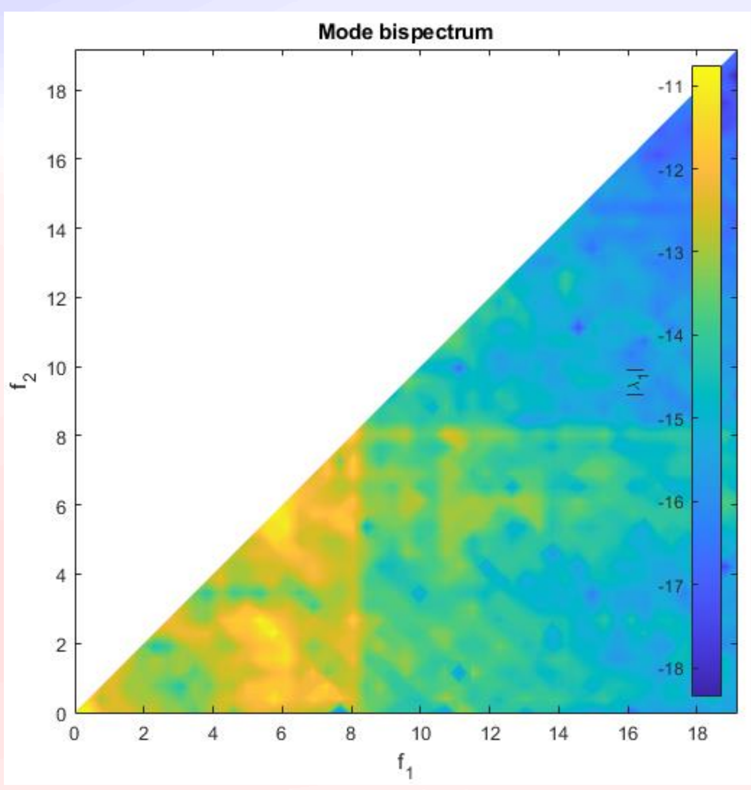


Figure 9: BMD complex mode bispectrum at α = 10°

Bi-spectral Modal Decomposition

- BMD educes coherent flow structures associated with triadic interactions from experimental or numerical data.
- It establishes a causal relationship between the three frequency components of a triad
- This permits the distinction of sum- and difference-interactions, and the computation of interaction maps indicating regions of nonlinear coupling.
- At α = 10°, many interactions are present between 4Hz and 8Hz, generating many new frequencies and adding to wake complexity.

Conclusions

- Both POD and DMD identified von Kármán vortex streets originating from shedding at the airfoil trailing edge.
- These structures contained the most energy at α = 0° and α = 10° and were represented by narrow frequency bands, due to vibration of the airfoil section during the data collection phase.
- Kelvin-Helmholtz instability was identified at α = 10° due to velocity shear between the boundary layer separation and main flow.
- Vortex streets and velocity shear were observed interacting through vortex merging at the α = 10° case. Multiple frequencies resulted from these combinations.
- Linking energy structures to their constituent frequencies allowed for identifications of flow features within POD, revealing the complexity of high-α flows.
- BMD confirmed that at high angles of attack, a greater number of interactions are present generating many new structures.
- Both techniques allowed for more meaningful exploitation of POD results by linking them to flow features known to exist around airfoils.

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